Multiple Choice

1. (5 pts.) An experiment consists of rolling two dice (say a red one and a green one) and recording the sum of the numbers that appear. Let E be the event that the sum is 6. Find $P(E)$.

(a) 5/36 (b) 6/36 (c) 3/36

(d) 2/36 (e) 4/36

Done in class. Look at all 36 options and only

$\underbrace{(5,1)}_{1}, \underbrace{(4,2)}_{2}, \underbrace{(3,3)}_{3}, \underbrace{(2,4)}_{4}, \underbrace{(1,5)}_{5}$ add up to 6.

Hence the answer is $5/36$.

2. (5 pts.) Suppose E and F are two events with $P(E) = 1/4$, $P(F) = 1/2$ and $P((E \cup F)') = 1/3$. Find $P(E \cap F)$.

(a) 1/8 (b) 1/12 (c) 1/4

(d) 1/6 (e) 1/3

$P(E \cup F) = 2/3$ why?

Use $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ to get

the answer.

3. (5 pts.) Three students are selected at random from a group of 12 boys and 9 girls. What is the probability that 2 of them are boys and the other one is a girl.

(a) $\frac{21}{9261}$ (b) $\frac{1188}{1330}$ (c) $\frac{594}{7980}$

(d) $\frac{1188}{7980}$ (e) $\frac{594}{1330}$

Done in Review class today - (March 5).

$$\frac{C(12,2)\,C(9,1)}{C(21,3)}$$

$\leftarrow$ $n(E)$ = # elements in our event which is selections with 2 boys and 1 girl.

$\leftarrow$ $n(S)$ = # elements in sample space.

4. (5 pts.) Five cards are randomly drawn from a bridge deck of cards (52 cards, 26 black, 26 red). What is the probability that at least one of them is red?

(a) $\frac{26^5}{52^5}$ (b) $\frac{65780}{2598960}$ (c) $\frac{9328800}{2598960}$ (d) $\frac{2533180}{2598960}$ (e) $\frac{26^5}{2598960}$

Done in review

E: at least one red

E': all black

$$P(E') = \frac{C(26,5)}{C(52,5)} = \frac{65780}{2598960}$$

$$P(E) = 1 - P(E') = \frac{2533180}{2598960}$$

5. (5 pts.) An experiment consists of flipping a coin 6 times and observing the sequence of heads and tails that occurs. Let E be the event there are (strictly) more heads than tails. Find $P(E)$.

(a) 18/64 (b) 21/64 (c) 22/64

(d) 44/64 (e) 32/64

E: outcomes with more heads than tails.
This can happen in 3 different ways:

Case 1: 6 H & 0 T	1 way in which this happens	Hence there are $1+6+15$ outcomes with more heads than tails. There are 2^6 total outcomes. So $\boxed{P(E) = \frac{22}{2^6} = \frac{22}{64}.}$
Case 2: 5 H & 1 T	$C(6,1) = 6$ " " "	
Case 3: 4 H & 2 T.	$C(6,2) = 15$ " " "	

6. (5 pts.) Let E and F be two events of an experiment. Which of the following statements is FALSE?

(a) if E and F are independent, then $P(E|F) = P(E)$

(b) if E and F are independent, then $P(E \cap F) = P(E)P(F)$

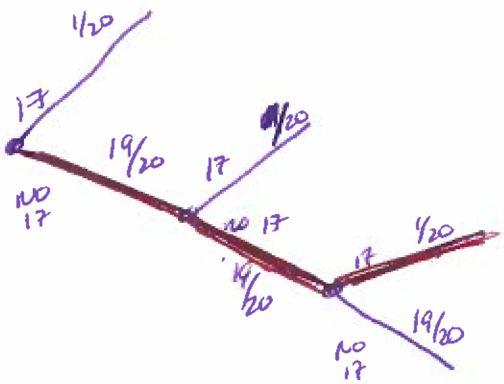
(c) if E and F are independent, then $P(F|E) = P(F)$

(d) If E and F are mutually exclusive, then $P(E \cup F) = P(E)P(F)$

(e) if E and F are mutually exclusive, then $E \cap F = \emptyset$

7. (5 pts.) On a new game show, The Dice is Trite, Claire is given a 20-sided die (with the sides labelled from 1 to 20, and all sides equally likely to come up). Each time she rolls it, if it is NOT a 17 she is given \$1,000 and told to roll again. When she rolls a 17, she is done. (For example, if a 17 appears on the fifth roll, she gets \$4,000 since she has \$1,000 for each of the first four rolls and nothing for the fifth roll.) What is the probability that she receives **exactly** \$2,000 [a tree diagram would probably help].

(a) $\left(\frac{19}{20}\right)^2 \frac{1}{20}$ (b) $\left(\frac{1}{20}\right)^3$ (c) $\frac{19}{20} \cdot \frac{1}{20}$ (d) $\left(\frac{19}{20}\right)^2$ (e) $\left(\frac{19}{20}\right)^2 + \frac{1}{20}$



I want "no 17" on first roll
and "no 17" on second roll
and "17" on third roll.

Hence answer is
$\left(\frac{19}{20}\right)\left(\frac{19}{20}\right)\left(\frac{1}{20}\right)$ (see red branch).

8. (5 pts.) Brian rolls a dice 2 times. Find the probability that he first rolls an even number and then a six.

(a) 5/12 (b) 1/2 (c) 1/6
(d) 1/12 (e) 1/2 + 1/6

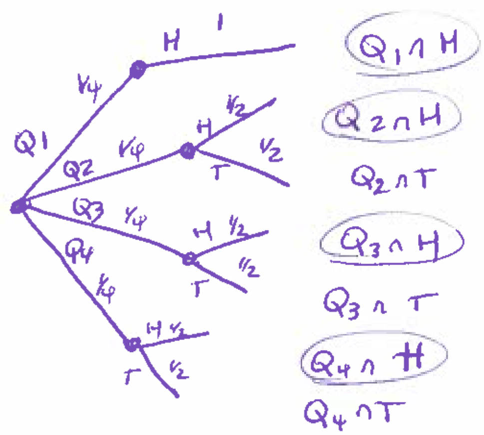
E: he rolls an even number
F: he rolls a 6.

$$P(E \cap F) = P(E)\,P(F|E)$$
$$= P(E)\,P(F)$$
$$= (3/6)(1/6)$$
$$= 3/36 = \boxed{1/12}$$

Notice each roll is independent.
Hence $P(F|E) = P(F)$

9. (5 pts.) Three ordinary quarters and a fake quarter with two heads are placed in a hat. One quarter is selected at random and flipped once. What is the probability that it comes up heads?

(a) 1/4 (b) 3/8 (c) 5/8 (d) 3/4 (e) 1/2



$(1/4)(1) + (1/4)(1/2) + (1/4)(1/2) + (1/4)(1/2)$

$= 5/8$

10. (5 pts.) At Grinnell College the number of students and of math majors divides as follows:

Class	No. Students	No. Math Majors
Freshmen	100	50
Sophomores	150	60
Juniors	200	70
Seniors	250	80
	700	260

Let F be the event that a randomly chosen student is a freshman, and M the event that a randomly chosen student is a math major. Find $P(F|M)$.

(a) $\frac{5}{26}$ (b) $\frac{5}{13}$ (c) $\frac{13}{35}$

(d) $\frac{1}{2}$ (e) $\frac{2}{3}$

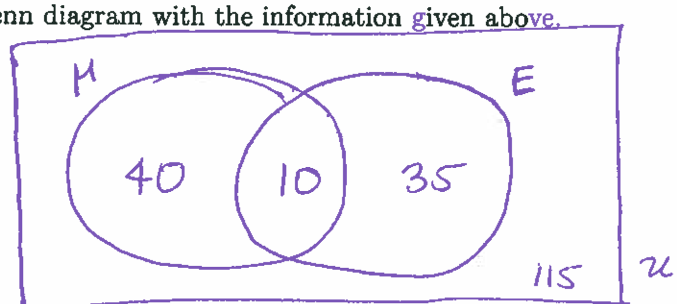
$$P(F|M) = \frac{P(F \cap M)}{P(M)} = \frac{50/700}{260/700} = \frac{50}{260} = \frac{5}{26}$$

Partial Credit

You must show **all of your work** on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) From a group of 200 students, 50 students are enrolled in Professor Mosby's architecture class, 45 students are enrolled in professor Eriksen's law class and 10 students are enrolled in both classes.

(a) Draw a Venn diagram with the information given above.



(b) A student is selected at random. Let M be the event "is enrolled in Prof. Mosby's class" and E be the event "is enrolled in Prof. Ericksen's class".
Are the events M and E mutually exclusive?

$\boxed{\text{M \& E are } \underline{\text{NOT}} \text{ mutually exclusive}}$ since

$M \cap E \neq \emptyset$. There are 10 students in $M \cap E$.

(c) Are the events M and E independent?

Lets verify whether $P(M \cap E) = P(M)P(E)$ or not.

$$P(M) = \frac{n(M)}{n(S)} = \frac{50}{200} = \frac{5}{20} = \frac{1}{4}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{45}{200} = \frac{9}{40}$$

$$P(M \cap E) = \frac{10}{200} = \frac{1}{20}.$$

Notice

$$P(M)P(E) = \left(\frac{1}{4}\right)\left(\frac{9}{40}\right) = \frac{9}{160}$$

So $\boxed{P(M)P(E) \neq P(M \cap E)}$

$\boxed{\text{M \& E are not independent}}$

12. (10 pts.) A pair of dice, one red and one blue, are rolled.
(a) What is the probability that the sum of the numbers facing up is 9?

E: results that add up to 9

S: any result of the roll.

$$P(E) = \frac{n(E)}{n(S)} = \cancel{\frac{3}{36}} = \boxed{\cancel{\frac{1}{12}}}$$

This should be 4/36 instead of 3/36.

Notice that there are 4 ways in which i cand get 9, namely:

4 (on red) and 5 (on blue)
5 (on red) and 4 (on blue)
6 (on red) and 3 (on blue)
3 (on red) and 6 (on blue)

(b) What is the probability that both numbers facing up are even.

Two ways of solving this

Way I: Look at all 36 outcomes and count the ones with both dice even. Not the ideal way of solving the problem.

Result is $\frac{9}{36}$.

Way 2:

Let E: getting an even # on red die.
F: getting an even # on blue die

Since the rolls are independent:

$$P(E \cap F) = P(E)P(F)$$
$$= (3/6)(3/6) = 9/36 = \boxed{\frac{1}{4}}$$

(c) Given that the number facing up in the red die is odd, what is the probability that the number facing up in the blue die is even.

E: # on red die is odd

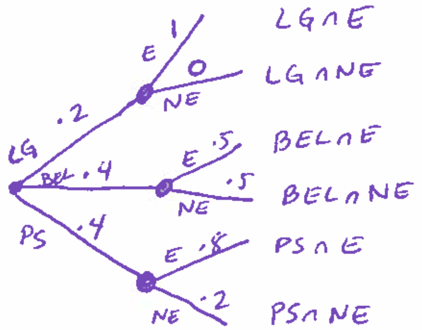
F: # on blue die is even

$$P(F/E) = P(F)$$
$$\boxed{= 3/6}$$

since when rolling 2 dice, each roll is independent to the other.

13. (10 pts.) Professor Bunsen always starts his Alchemy 101 lecture course with one of the three great alchemical experiments: turning lead into gold (20% of all times that he teaches the course), brewing the elixir of life (40% of the times) and creating the Philosopher's stone (40% of the time). When he tries to turn lead into gold, the result always ends with a explosion; when he brews the elixir of life, there is a 50% chance of an explosion, and when he creates the Philosopher's stone, 8 times out of 10 there is an explosion.

(a) The next time Professor Bunsen teaches the course, what is the probability of an explosion happening?



$$P(E) = P(LG \cap E) + P(BEL \cap E) + P(PS \cap E)$$
$$= (.2)(1) + (.4)(.5) + (.4)(.8)$$
$$= .72$$

(b) What is the probability that either there is an explosion, or the professor attempts to brew the elixir of life?

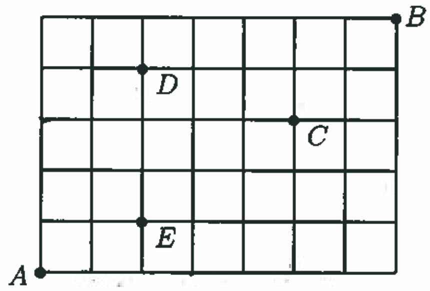
$$P(E \cup BEL) = P(E) + P(BEL) - P(E \cap BEL)$$
$$= .72 + (.4) - (.4)(.5)$$
$$\boxed{= .92}$$

(c) Dean Crawford wants to see which experiment Professor Bunsen will do this year, but he arrives late. If he see the lecture-hall filled with post-explosion smoke, what (should he conclude) is the probability that he has just missed a demonstration of brewing the elixir of life?

$$P(BEL \mid E) = \frac{P(BEL \cap E)}{P(E)} = \frac{(.4)(.5)}{(.2)(1) + (.4)(.5) + (.4)(.8)}$$

$$\boxed{\approx .277}$$

14. (10 pts.) This question involves the usual city grid, with bottom left point A, top right point B, and three middle points C, D, E, with D and E having the same x-coordinate, D having a higher y-coordinate, and C have a greater x-coordinate than both D and E, and a y-coordinate that is in between. The story is that John travels from A to B, using as few blocks as possible, choosing his particular route randomly (all routes equally likely).



(a) What's the probability that along the way he passes C?

E: paths from A to B, passing through C.

S: paths from A to B.

$$P(E) = \frac{C(8,5)\cdot C(4,2)}{C(12,7)} = \boxed{\frac{14}{33}} \approx \boxed{.4242}$$

(after simplifying)

(b) At some point he is seen passing E. Now what's the probability that along the way he passes C?

E: paths from A to B passing through C.

F: paths from A to B passing through E.

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{C(3,2)\,C(5,3)\,C(4,2)}{C(12,7)}}{\frac{C(3,2)\,C(9,5)}{C(12,7)}} = \frac{5}{22} \boxed{\approx .2272}$$

(c) At some point he is seen passing D. Now what's the probability that along the way he passes C?

$\boxed{0}$ If he is seen at D then he did not passed through C since C is below D and he goes up or right.

15. (10 pts.) A child has 6 cards numbered $\{2, 3, 4, 5, 6, 7\}$. The child places three cards in a row to create a 3-digits number.
(a) What is the probability that the number selected is 374?

E: selection 374
S: all selections

$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6 \cdot 5 \cdot 4}$

$\boxed{= \frac{1}{120}}$

(b) What is the probability that the number selected is smaller than 500?

E: selections with # smaller than 500.
S: all selections.

$\underset{2,3 \text{ or } 4}{3} \cdot 5 \cdot 4$

$P(E) = \frac{n(E)}{n(S)} = \frac{60}{120} = \boxed{\frac{1}{2}}$

(c) What is the probability that the number selected is bigger than 550?

E: selections with # bigger than 550.

If I don't divide it into cases, what happens?

$\underset{5, 6 \text{ or } 7}{\_\_}$ $\_\_$ $\_\_$

depends on whether 1st is a 5 or a 6,7.

Case 1: 1st # is a 5, then there are $\boxed{8}$ numbers. $\underset{5}{1} \cdot \underset{6,7}{2} \cdot 4$

Case 2: 1st # is 6, or 7, then there are $\boxed{40}$ numbers. $2 \cdot 5 \cdot 4$

$P(E) = \frac{n(E)}{n(S)} = \frac{\boxed{48}}{120}$.

(d) What if the probability that the number selected is between 500 and 550?

Two ways:

Way 1:

E: selections with # between 500 and 550.

$\underset{\text{has to be a 5}}{1} \cdot \underset{\text{has to be 2,3 or 4}}{3} \cdot \underset{\text{anything else.}}{4}$

$P(E) = \frac{n(E)}{n(S)} = \frac{12}{120} = \boxed{\frac{1}{10}}$

Way 2:

E: selections with # between 500 & 550.
E': selections with # $\underbrace{\text{less than 500}}_{A}$ OR $\underbrace{\text{bigger than 550}}_{B}$.

$P(E') = P(A \cup B) = \underbrace{P(A)}_{\text{part (b)}} + \underbrace{P(B)}_{\text{part (c)}} - \cancel{P(A \cap B)}$ 0 why? $A \cap B = \emptyset$

$= \frac{1}{2} + \frac{48}{120} = \frac{108}{120}$.

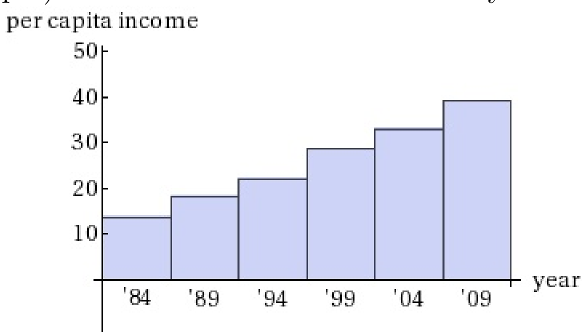
$P(E) = 1 - P(E') = 1 - \frac{108}{120} = \frac{12}{120} = \boxed{\frac{1}{10}}$

1.(10pts) A mathematics quiz consists of 5 questions. There are 75 students in the class. Below is a frequency table showing how the class did.

Question	**# correct answers**
1	36
2	41
3	22
4	54
5	30

If I select 25 students at random from the class approximately how many got problem 3 correct?

2.(6pts) Here are the results of a survey done every 5 years of per capita income.



Which of the following statements can deduced from the information given in the histogram?

(a) In 2009 at least one person had an income of at least 40.

(b) At least one person's income rose steadily from 1986 to 2009.

(c) No one in 1996 made 40.

(d) The number of people making 40 in 2009 is greater than the number of people making 30 a year in 1999

3.(10pts) Make a stem-and-leaf plot for the given scores.

$$31, 22, 40, 24, 41, 33, 26, 35, 36, 49, 28, 25, 32, 42, 37, 47, 20, 38, 44, 37, 21, 41$$

1. Solution. $\dfrac{22 \cdot 25}{75} = \dfrac{22}{3}$. If it were multiple choice I'd pick either 7 or 8.

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2. Solution. If everyone in 2009 made less than 40 a year the per capita income could not be 40.

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3. Solution.

Stem	**Leaves**
2	0 1 2 4 5 6 8
3	1 2 3 5 6 7 7 8
4	0 1 1 2 4 4 7 9
